Liquid Crystals
Publication details, including instructions for authors and subscription information:
http://www.informaworld.com/smpp/title~content=t713926090

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To cite this Article Tsoy, V. I. , Simonenko, G. V. and Chigrinov, V. G.(1993) 'Dielectric stripes in pretilted supertwisted layers', Liquid Crystals, 13: 2, 227 — 231
To link to this Article: DOI: 10.1080/02678299308026296
URL: http://dx.doi.org/10.1080/02678299308026296

Liquid Crystals, 1993, Vol. 13, No. 2, 227–231

Dielectric stripes in pretilted supertwisted layers

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(*Received 16 March 1992; accepted 11 July 1992*)

The modes of both the director configuration and the electrical field in dielectric domain stripes are found for a pretilted supertwisted nematic layer. Good agreement between the calculated data and the reported experimental data is obtained. The domain voltage range as well as the threshold voltage can be calculated.

1. Introduction

The physical causes and the main factors for the formation of dielectric stripes in twisted layers was revealed long ago [1–5]. In the non-trivial case of a small Grandjean zone number the domain threshold voltage was first calculated numerically in [2]. The supertwisted display is just in this regime but the domain stripes disturb the display [4]. However, the computer simulation for these domains has been carried with two restrictive assumptions; the first is a homogeneous electric field, the second is an initial non-pretilted director. In actual fact a more detailed simulation is needed to obtain valid domain range parameters. The first of these restrictions was taken into account in [3], the second is considered here.

2. Director and electric field configuration

We consider a cholesteric layer between planes at $z = \pm L/2$ with a director $\mathbf{n}$ tilted by an angle $\theta(z)$ with respect to the layer and $\phi(z)$ is the azimuthal angle for the director with respect to the x axis. Thus the director is $(\cos\theta\cos\phi, \cos\theta\sin\phi, \sin\theta)$. The dimensionless coordinates and derivatives are $\kappa = x/L$, $\eta = y/L$, $\zeta = z/L$, $\mathrm{Div} = L\,\mathrm{div}$, $\mathrm{Rot} = L\,\mathrm{rot}$. Let us assume that the κ axis is in the same direction as the domain stripes. Then an electric field satisfies the equations [3]

$$E_\kappa = 0, \tag{1}$$

$$\mathrm{Rot}\,\mathbf{E} = \partial E_\zeta/\partial\eta - \partial E_\eta/\partial\zeta = 0, \tag{2}$$

$$\mathrm{Div}\,\tilde{\varepsilon}\,\mathbf{E} = \frac{\partial}{\partial\eta}(\varepsilon_{yy}E_\eta + \varepsilon_{yz}E_\zeta) + \frac{\partial}{\partial\zeta}(\varepsilon_{zy}E_\eta + \varepsilon_{zz}E_\zeta) = 0, \tag{3}$$

$$\int_{-1/2}^{1/2} E_\zeta\, d\zeta = U/L, \tag{4}$$

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where

$$\varepsilon_{yy}=\varepsilon_{\perp}+\Delta\varepsilon\cos^2\theta\sin^2\varphi,\quad \varepsilon_{yz}=\varepsilon_{zy}=\Delta\varepsilon\sin\theta\cos\theta\sin\varphi,$$
$$\varepsilon_{zz}=\varepsilon_{\perp}+\Delta\varepsilon\sin^2\theta$$

are dielectric tensor components;

$$\Delta\varepsilon=\varepsilon_{\parallel}-\varepsilon_{\perp}$$

is the dielectric anisotropy; $\varepsilon_{\parallel}$, $\varepsilon_{\perp}$ are dielectric permeability components parallel and perpendicular to the director; U is the control voltage.

Under strong liquid crystal surface anchoring the equilibrium director configuration corresponds to a volume free energy variation of zero

$$\delta\mathbf{F}=\delta\int_0^{1/q}d\eta\int_{-1/2}^{1/2}d\zeta[k_{11}/2(\mathrm{Div}\,\mathbf{n})^2+k_{22}/2(\mathbf{n}\,\mathrm{Rot}\,\mathbf{n}+2\pi L/p)^2$$
$$+k_{33}/2(\mathbf{n}\times\mathrm{Rot}\,\mathbf{n})^2-L^2/(8\pi)(\varepsilon_{yy}E_\eta^2+2\varepsilon_{yz}E_\eta E_\zeta+\varepsilon_{zz}E_\zeta^2)]=0, \quad (5)$$

where $q=L/\Lambda$ is a dimensionless stripe spatial frequency; Λ is a spatial period; k_{11}, k_{22}, k_{33} are elastic constants for the director field deformation and p is the pitch of the helix. In the case of $\partial/\partial\eta=0$, $E_\eta=0$ equations (1)–(5) describe a one-dimensional plane homogeneous configuration $\{\theta_s(\zeta),\varphi_s(\zeta),E_{\zeta s}(\zeta)\}$. Such a free domain state will be disturbed if a periodic structure along the η axis satisfies equation (5). The periodic configuration compatible with equations (1)–(4) has the form

$$\theta(\eta,\zeta)=\theta_s(\zeta)+\delta[V_\theta(\zeta)C_q+W_\theta(\zeta)S_q], \quad (6)$$

$$\varphi(\eta,\zeta)=\varphi_s(\zeta)-\alpha+\delta[V_\varphi(\zeta)C_q+W_\varphi(\zeta)S_q], \quad (7)$$

$$E_\zeta(\eta,\zeta)=E_{\zeta s}(\zeta)+\frac{U}{L}\delta[F'(\zeta)C_q+G'(\zeta)S_q], \quad (8)$$

$$E_\eta(\eta,\zeta)=2\pi q\frac{U}{L}\delta[G(\zeta)C_q-F(\zeta)S_q], \quad (9)$$

where $C_q=\cos(2\pi q\eta)$; $S_q=\sin(2\pi q\eta)$; α is the angle between the domain stripes and a mean director azimuth from those at the surfaces while the conditions $\phi_s(\pm1/2)=\pm\Phi_T/2$ are valid. The Φ_T value is a total twisted angle. The small parameter δ gives the perturbation of the preliminary one dimensional configuration at the same voltage U.

Equations (8) and (9) already satisfy the condition (2); from (3) we obtain, to first order in δ, the differential equations for the amplitudes $F(\zeta)$, $G(\zeta)$ of an electric field in domains in the form

$$A_1F''+A_2F'-A_3F+A_4G'+A_5G+A_6V'_\theta+A_7W_\theta+A_8W_\varphi=0, \quad (10)$$

$$A_1G''+A_2G'-A_3G-A_4F'-A_5F+A_6W'_\theta-A_7V_\theta-A_8V_\varphi=0, \quad (11)$$

where

$$A_1=\frac{U}{L}(\varepsilon_\perp+\Delta\varepsilon S_\theta^2),$$

$$A_2=\frac{U}{L}\Delta\varepsilon S_{2\theta}\theta'_s,$$

$$A_3=(2\pi q)^2\frac{U}{L}(\varepsilon_\perp+\Delta\varepsilon S_\varphi^2 C_\theta^2),$$

$$A_4=2\pi q\frac{U}{L}\Delta\varepsilon S_{2\theta}S_\varphi,$$

$$A_5=2\pi q\frac{U}{L}\Delta\varepsilon(C_{2\theta}S_\varphi\theta'_s+S_\theta C_\theta C_\varphi\varphi'_s),$$

$$A_6=E_{\zeta s}\Delta\varepsilon S_{2\theta},$$

$$A_7=2\pi q E_{\zeta s}\Delta\varepsilon C_{2\theta}S_\varphi,$$

$$A_8=2\pi q E_{\zeta s}\Delta\varepsilon S_\theta C_\theta C_\varphi,$$

$$C_x=\cos x;\ S_x=\sin x.$$

After the simple but tedious procedure with equations (5)–(7) we obtain the differential equations for the amplitudes of the director angles in domains in the form

$$V''_\theta+B_1/(k_{11}C_\theta^2+k_{33}S_\theta^2)=0, \tag{12}$$

$$W''_\theta+B_2/(k_{11}C_\theta^2+k_{33}S_\theta^2)=0, \tag{13}$$

$$V''_\varphi+B_3/\{C_\theta^2(k_{22}C_\theta^2+k_{33}S_\theta^2)\}=0, \tag{14}$$

$$W''_\varphi+B_4/\{C_\theta^2(k_{22}C_\theta^2+k_{33}S_\theta^2)\}=0. \tag{15}$$

Here the B_1, B_2, B_3, B_4 values are given by rather complication formulae which contain V_θ, W_θ, V_φ, W_φ, and the amplitudes F, G, their first derivatives, director angles, the electric field strength in an initial one dimensional configuration, helix pitch, elastic constants, dielectric constants, azimuth and period of the stripes, together with the control voltage. As a result the stable periodic configuration has to be found from six second-order differential equations (10)–(15).

Lack of the tangential field at the equipotential electrodes on surfaces means that domains do not disturb the field values at the layer boundaries. Therefore, the solution of equations (10)–(15) satisfies the boundary conditions

$$F(\pm\tfrac{1}{2})=G(\pm\tfrac{1}{2})=V_\theta(\pm\tfrac{1}{2})=W_\theta(\pm\tfrac{1}{2})=V_\varphi(\pm\tfrac{1}{2})=W_\varphi(\pm\tfrac{1}{2})=0. \tag{16}$$

The problem in equations (10)–(16) describes the appearance of domain stripes for an arbitrary director tilt at the surfaces including the case of unequal angles. In particular, for zero pretilt director $\theta_s(\zeta)=G(\zeta)=V_\theta(\zeta)=W_\varphi(\zeta)=0$ and the six equations (10)–(15) are reduced to three investigated in [3]. Moreover, only two equations remain if the dielectric anisotropy is small as in [2].

3. Domain excitation conditions

As shown in [2] appearance of field domains in a cholesteric layer occurs if the control voltage is greater then the threshold. In addition there is a spatial dispersion of the threshold voltage. The numerical procedure reported in [2] which gives the dispersion curve $U(q)$ remains in general the same while director pretilt is non-zero. Only a slight complication results because of increasing the number of equations as well as the necessity to calculate the one dimensional configuration under current voltage. We now denote the V_θ, W_θ, V_φ, W_φ, F, G perturbation amplitudes by odd $Y_{2k-1}(\zeta)$ components of a twelve dimensional vector $Y(\zeta)$. Also the amplitude derivatives are denoted by the even components $Y_{2k}(\zeta)$. Therefore the problem consists of the solution of a system of first-order differential equations with respect to $Y(\zeta)$, with boundary conditions $Y_{2k-1}(\pm\frac{1}{2})=0$. In order to satisfy the boundary conditions at $\zeta=-\frac{1}{2}$ the solution has to take the form

$$Y_i(\zeta)=\sum_{r=1}^{6} R_r Y_{i,r}, \tag{17}$$

where $\{Y_{i,r}\}$ is the set of six fundamental solutions calculated under the initial conditions $Y_{i,r}(-\frac{1}{2})=\delta_{i,2r}$. Then the boundary conditions at $\zeta=\frac{1}{2}$ provide the equations (17) for the R_r coefficients. So the determinant of the matrix $Y_{2k-1}(\frac{1}{2})$ is zero:

$$\mathrm{Det}\,(Y_{2k-1,r}(\tfrac{1}{2}))=\mathrm{Det}\,(U,q)=0. \tag{18}$$

This equation allows us to obtain, in the (U,q) plane, the curve separating domain and free-domain ranges. An example is shown in figure 1 for a transverse domain ($\alpha=90°$) in a layer with a twist angle Φ_T of 200° and elastic constants $k_{11}=8{\cdot}8\times10^{-12}$ N, $k_{22}=5{\cdot}04\times10^{-12}$ N, $k_{33}=12{\cdot}6\times10^{-12}$ N, the thickness–pitch ratio was $L/p=0{\cdot}61$, the dielectric constants were $\varepsilon_\parallel=14{\cdot}1$, $\varepsilon_\perp=4{\cdot}1$. Curves 1, 2, 3 are obtained for the non-pretilted director configuration. Curve 1 corresponds to the equations from [2] while curve 2 comes from [3]. We can see that the dielectric anisotropy taken into account in curve 2 affects the result of the calculation. Curve 3 is obtained in the framework of the current article. Provided the control voltage is less than the threshold curve 3 coincides with curve 2. If the voltage is greater than the threshold then curve 3 closes and becomes a boundary for a range in which the domain configuration is stable. Curve 4 corresponds to the pretilted director configuration with $\theta(\pm1/2)=3°$. One can see that increasing the pretilt angle induces the narrowing of the domain range. Thus a pretilt angle increases the domain threshold and decreases the domain suppressing voltage. In addition the levels of both Freedericksz transition voltage U_F and Breddels point U_B are shown. It can be noted that the domain voltage range may be rather far from Breddels point (θ_B=constant) as in figure 1. Therefore the conclusion from [4] concerning Breddels point is not always valid.

4. Discussion

The experimental data for the ZLI 2293 mixture from [4] are shown in figure 2 as curve 1. This curve represents the critical L/p ratio for domain formation as a function of the pretilt angle θ. For comparison curve 2 calculated in the framework of this article is also shown. We can see good agreement between the experimental data and the theoretical curves. It seems that proposed simulating procedure should be useful for investigating and designing supertwisted displays.

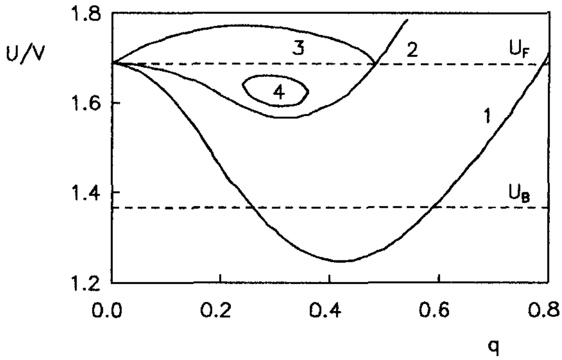

Figure 1. Relationship between the domain critical voltage U and the normalized spatial frequency q.

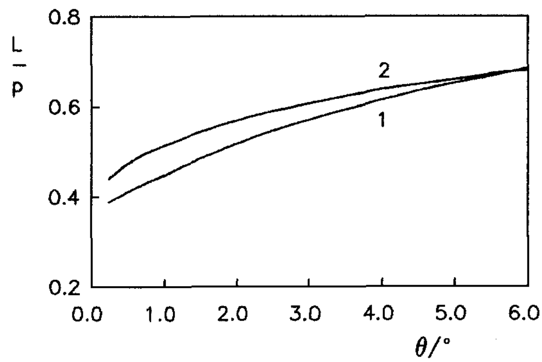

Figure 2. Relationship between the pretilt angle θ and the critical thickness–pitch ratio L/p.

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